University of California, Berkeley
Physics 105 Fall 2000 Section 2 (*Strovink*)

ASSIGNMENT 9

Reading:
105 Notes 10.1-10.3, 11.1-11.4
Hand & Finch 8.4-8.12

1.
Three equal point masses are located at $(a, 0, 0)$, $(0, 2a, 2a)$, and $(0, 2a, a)$. About the origin, find the principal moments of inertia and a set of principal axes.

2.
Consider a rigid body that is plane, *i.e.* it lies in the plane $z = 0$.
(**a**)
Prove that the z axis is a principal axis.
(**b**)
Prove that the diagonalized inertia tensor for this plane rigid body has the largest element equal to the sum of the two smaller elements.

3.
Design a solid right circular cylinder so that if it is rotated about *any* axis that passes through its center of mass, it will continue to rotate about that axis without wobbling.

4.
Assume that the earth is a rigid solid sphere that is rotating about an axis through the North Pole. At $t = 0$ a mountain of mass 10^{-9} the earth's mass is added at north latitude 45°. The mountain is added "at speed" so that the earth's angular velocity ω is the same before and immediately after the mountain's addition.

Describe the subsequent motion of the rotation axis with respect to the North Pole. What is the velocity of its intersection with the earth's surface, in miles per year?

5.
Assume that the earth is a rigid solid ellipsoid of revolution, rotating about its symmetry axis $\hat{\mathbf{x}}_3$, and that it has $1 - (I_2/I_3) = -0.0033$ (actually the earth *bulges* at the equator, so that this quantity is really positive). Two equal mountains are placed opposite each other on the equator "at speed", so that ω is the same immediately afterward. What fraction of the earth's mass must each mountain have in order to render the earth's rotation barely unstable with respect to small deviations of $\hat{\omega}$ from the $\hat{\mathbf{x}}_3$ axis?

6.
Consider an asymmetric body (principal moments $I_3 > I_2 > I_1$) initially rotating with $\vec{\omega}$ very close to the $\hat{x}_3$ axis.
(**a**)
Show that the projection of $\vec{\omega}(t)$ on the $\hat{x}_1 - \hat{x}_2$ plane describes an *ellipse*.
(**b**)
Calculate the ratio of the major and minor axes of the ellipse.

7.
Consider a heavy symmetrical top with one point fixed. Show that the *magnitude* of the top's angular momentum about the fixed point can be expressed as a function only of the constants of motion and the polar angle θ of the top's axis.

8.
Investigate the motion of the heavy symmetrical top with one point fixed for the case in which the axis of rotation is vertical (along $\hat{\mathbf{x}}_3$). Show that the motion is either stable or unstable depending on whether the quantity $4I_2Mhg/I_3^2\omega_3^2$ is less than or greater than unity. (If the top is set spinning in the stable configuration ("sleeping"), it will become unstable as friction gradually reduces the value of ω_3. This is a familiar childhood observation.)